

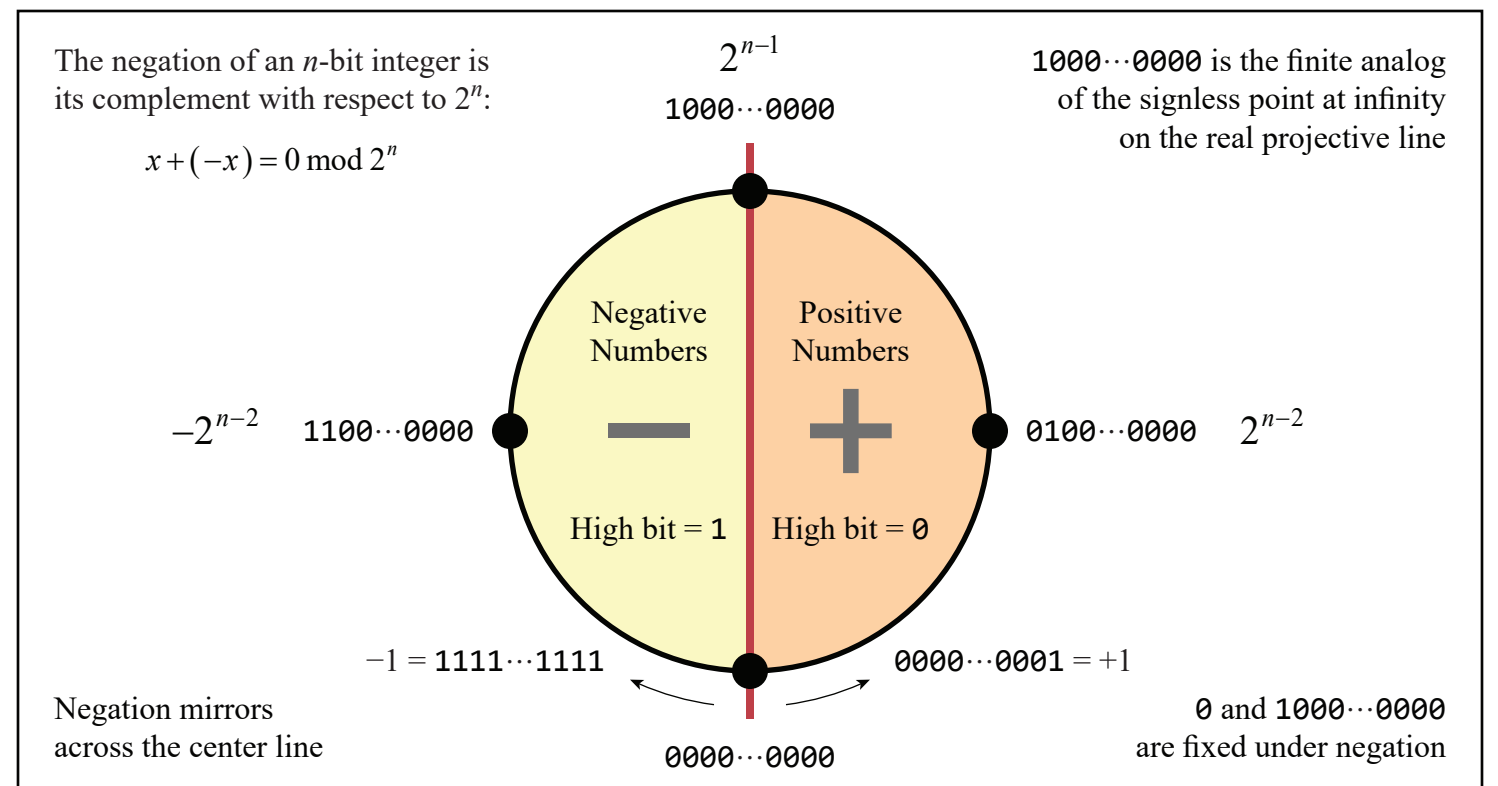
Binary Fundamentals

Powers of Two

Power	Decimal	Hexadecimal
2^1	2	0x00000002
2^2	4	0x00000004
2^3	8	0x00000008
2^4	16	0x00000010
2^5	32	0x00000020
2^6	64	0x00000040
2^7	128	0x00000080
2^8	256	0x00000100
2^9	512	0x00000200
2^{10}	1024	0x00000400
2^{11}	2048	0x00000800
2^{12}	4096	0x00001000

Power	Decimal	Hexadecimal
2^{13}	8192	0x00002000
2^{14}	16,384	0x00004000
2^{15}	32,768	0x00008000
2^{16}	65,536	0x00010000
2^{17}	131,072	0x00020000
2^{18}	262,144	0x00040000
2^{19}	524,288	0x00080000
2^{20}	1,048,576	0x00100000
2^{21}	2,097,152	0x00200000
2^{22}	4,194,304	0x00400000
2^{23}	8,388,608	0x00800000
2^{24}	16,777,216	0x01000000

Two's Complement



Logical Complement

NOT
Bitwise NOT
 $\sim x$

x	$\sim x$
0	1
1	0

Logical Identities

Unary	Binary
$\sim x = -x - 1$	$\sim(x \& y) = \sim x \mid \sim y$
$-x = \sim x + 1$	$\sim(x \mid y) = \sim x \& \sim y$
$\sim \sim x = x + 1$	$\sim(x \wedge y) = \begin{cases} \sim x \wedge y \\ x \wedge \sim y \end{cases}$
$\sim \sim x = x - 1$	

Floating-Point

Half precision	Single precision		
16-bit floating-point	32-bit floating-point		
 sign s value $= (-1)^s 2^{e-15} \left(1 + \frac{m}{2^{10}}\right)$	 sign s value $= (-1)^s 2^{e-127} \left(1 + \frac{m}{2^{23}}\right)$		
Double precision			
64-bit floating-point			
 sign s value $= (-1)^s 2^{e-1023} \left(1 + \frac{m}{2^{52}}\right)$			
Special Floating-Point Value	Half	Float	Double
+0.0	0x0000	0x00000000	0x00000000_00000000
+1.0	0x3C00	0x3F800000	0x3FF00000_00000000
Positive infinity	0x7C00	0x7F800000	0x7FF00000_00000000
Smallest positive normalized value	0x0400	0x00800000	0x00100000_00000000
Upper limit of non-integer values	0x6400	0x4B000000	0x43300000_00000000
Largest representable positive value	0x7BFF	0x7FFFFFFF	0x7FEFFFFFFF_FFFFFFFF

Binary Logical Operations

AND Bitwise AND $x \& y$ <table border="1"> <thead> <tr><th>x</th><th>y</th><th>$x \& y$</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	$x \& y$	0	0	0	0	1	0	1	0	0	1	1	1	OR Bitwise OR $x \mid y$ <table border="1"> <thead> <tr><th>x</th><th>y</th><th>$x \mid y$</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	$x \mid y$	0	0	0	0	1	1	1	0	1	1	1	1
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Mask Creation

Formula	Operation / Effect	Illustration
$\sim x \mid (x - 1)$	Create mask for all bits other than lowest 1 bit. $000 \dots 000$ becomes $111 \dots 111$.	
$x \mid \sim(x + 1)$	Create mask for all bits other than lowest 0 bit. $111 \dots 111$ is unchanged.	
$x \mid -x$	Create mask for bits left of lowest 1 bit, inclusive. $000 \dots 000$ is unchanged.	
$x \wedge -x$	Create mask for bits left of lowest 1 bit, exclusive. $000 \dots 000$ is unchanged.	
$\sim x \mid (x + 1)$	Create mask for bits left of lowest 0 bit, inclusive. $111 \dots 111$ becomes $000 \dots 000$.	
$\sim x \wedge (x + 1)$	Create mask for bits left of lowest 0 bit, exclusive. $111 \dots 111$ becomes $000 \dots 000$.	
$x \wedge (x - 1)$	Create mask for bits right of lowest 1 bit, inclusive. $000 \dots 000$ becomes $111 \dots 111$.	
$\sim x \& (x - 1)$	Create mask for bits right of lowest 1 bit, exclusive. $000 \dots 000$ becomes $111 \dots 111$.	
$x \wedge (x + 1)$	Create mask for bits right of lowest 0 bit, inclusive. $111 \dots 111$ is unchanged.	
$x \& (\sim x - 1)$	Create mask for bits right of lowest 0 bit, exclusive. $111 \dots 111$ is unchanged.	

Bit Manipulation

Formula	Operation / Effect	Illustration
$x \& (x - 1)$	Clear lowest 1 bit. If result is zero, then x is zero or 2^k . $000 \dots 000$ is unchanged.	
$x \mid (x + 1)$	Set lowest 0 bit. $111 \dots 111$ is unchanged.	
$x \mid (x - 1)$	Set all bits to right of lowest 1 bit. $000 \dots 000$ becomes $111 \dots 111$.	
$x \& (x + 1)$	Clear all bits to right of lowest 0 bit. If result is zero, then x is zero or $2^k - 1$. $111 \dots 111$ becomes $000 \dots 000$.	
$x \& -x$	Extract lowest 1 bit. $000 \dots 000$ is unchanged.	
$\sim x \& (x + 1)$	Extract lowest 0 bit (as a 1 bit). $111 \dots 111$ becomes $000 \dots 000$.	